Topic Study Group 32

Knowledge in/for Teaching Mathematics at Primary Level

Stéphane Clivaz[1], Kam Ling Lao[2], Janne Fauskanger[3], and Verónica Martín-Molina[4]

1. Themes and Description

Following Shulman's (1986) suggestion that teaching requires knowledge that is distinctive of the teaching profession, teachers' knowledge in/for teaching mathematics has attracted researchers worldwide. Different approaches have emerged on how this knowledge can be studied, developed, and strengthened and Topic Study Group 32 (TSG-32) has extended this conversation, with a focus on several emerging issues. TSG-32 at ICME 14 invited paper submissions on significant (new) trends and developments in research, theory, and practice about all different aspects that relate to the knowledge in/for teaching mathematics at primary level (learners' ages 5–13).

The following (often partly overlapping) themes have been considered.

(1) Focus on children's mathematics
- How attending to children's mathematical thinking can influence teachers' knowledge
- How elementary teachers use the leverage of core mathematical knowledge to nurture children's mathematical minds

(2) Focus on teacher learning
- Learning in collaborative communities (e.g., professional learning communities, lesson study, etc.)
- Acquisition of mathematical knowledge in teacher training
- Learning through teaching

(3) Focus on various aspects and uses of knowledge in/for teaching mathematics
- Cultural aspects such as cultural responsiveness, equitable teaching, teaching of mathematics in social and political contexts, etc.

[1]Mathematics and Science Education Department, Lausanne University of Teacher Education, 1014 Lausanne, Switzerland. E-mail: stephane.clivaz@hepl.ch

[2]School of Education and Languages, Hong Kong Metropolitan University, Hong Kong-China. E-mail: pollylao@gmail.com

[3]University of Stavanger, Faculty of Arts and Education, Department of Education and Sports Science, Norway. E-mail: janne.fauskanger@uis.no

[4]Departamento de Didáctica de las Matemáticas, Universidad de Sevilla, Sevilla 41013, Spain. E-mail: veronicamartin@us.es

- Aspects of teaching practice, such as high-levering/ambitious practices, attention to diverse learners, etc.
- Use of mathematical knowledge in the different phases of teaching (lesson planning, observation of students, task design, situations of contingency, etc.)

(4) Focus on methods for studying mathematical knowledge in/for teaching

2. Program Overview

TSG-32 had 3 sessions with presentations of papers and discussions of them. 26 papers and posters were submitted to TSG-32 and 3 papers were invited, resulting in the acceptance of 2 invited talks (IT), 8 long oral presentations (LO), and 8 short oral presentations (SO). However, several authors did not attend the TSG-32 sessions, probably due to the Covid pandemic and postponement of ICME 14 and/or because of the online mode of the conference. There was also a joint poster session, and 4 posters were accepted. Tab. 1 includes the 10 papers and 2 posters that were actually presented during the conference.

Tab. 1. Papers and posters presented at TSG-32

Paper and author(s)	
[1]	Seeing mathematics through the lens of children's mathematical thinking: a perspective on the enhancement of mathematical knowledge for teaching. ***Randolph A. Philipp**, John Siegfried, and Eva Thanheinser* (USA). (IT)
[2]	Towards a dialogic analysis of mathematical problem-solving knowledge for teaching in a lesson study group. ***Stéphane Clivaz**, Valérie Batteau, Audrey Daina, Luc-Olivier Bunzli, and Sara Presutti* (Switzerland). (LO)
[3]	Exploring preservice teachers' noticing of resources that support productive struggle and promote equity. *Christine Alyssa Herrera, Shawnda Rae Smith, Christina Starkey, and* ***Hiroko Kawaguchi Warshauer*** (USA). (LO)
[4]	A comparative study on the professional knowledge of elementary mathematics teachers in Shanghai and Hong Kong — from two scenarios in data handling and geometry. ***Kam Ling Lao*** (Hong Kong SAR, China). (LO)
[5]	Primary teachers' recognition of students' mathematical reasoning and beliefs about teaching and learning. ***Carolyn A. Maher**, James A. Maher, and* ***Louise Cherry Wilkinson*** (USA). (LO)
[6]	Pre-service primary teachers' knowledge and the mathematical practice of defining. ***Verónica Martín-Molina*** (Spain). (LO)
[7]	Teacher time out as site for studying mathematical knowledge for teaching. *Reidar Mosvold,* ***Janne Fauskanger**, Kjersti Wæge, and Raymond Bjuland* (Norway). (LO)
[8]	Addition and multiplication teaching in the multi-grade primary school. ***Yolanda Chávez Ruiz*** *and* ***Lorena Trejo Guerrero*** (Mexico). (SO)
[9]	Elementary preservice teachers' expected challenges in teaching pattern generalization. ***Mi Yeon Lee*** *and Ji-Eun Lee* (USA). (SO)
[10]	Why does 1/4:1/5 equal 5/4? A case of a post-graduate student's understanding of common fractions division. ***Barbara Beata Pieronkiewicz*** (Poland). (SO)
[11]	Unpacking performance indicators in the TPACK (Technological Pedagogical Content Knowledge) levels rubric to examine differences in the TPACK levels for teaching mathematics in primary schools. *Aleksandra Kaplon-Schilis and* ***Irina Lyublinskaya*** (USA). (Poster)
[12]	Developing analytical models of pedagogical content knowledge: a case study of mathematics teachers in Macao. ***Huey Lei*** (Macao SAR, China). (Poster)

Despite the difficulty of attending some sessions for many participants, probably due to the time difference, the discussions were of high quality.

3. Future Directions and Suggestions

Some of the papers presented in TSG-32 offered suggestions for future directions. Firstly, more studies are needed to determine how teachers develop knowledge in/for teaching mathematics in collaborative settings, which is very complex. Secondly, it would be interesting to study teachers' beliefs about the importance of their students' conceptual understanding of the mathematics that they should learn in school, or how that conceptual understanding could be promoted. While there is a need to investigate the diverse mathematical knowledge base among in-service teachers, there is also much to be discovered concerning how to help pre-service primary teachers to acquire an appropriate level of mathematical content knowledge. Focusing on mathematics through the lens of mathematical thinking could be used to help teachers (since their view of their students would become "richer and more nuanced"), but how to implement this in teacher training programs remains to be seen.

In their review of studies of mathematical knowledge for teaching, Hoover et al. (2016) identified 190 studies published between 2006 and 2013. Based on this review, they suggested that "a central problem for progress in the field is a lack of clearly understood and practicable methodology for the study and development of mathematical knowledge for teaching" (Hoover et al., 2016, p. 20). They further argue that the use of measures and interviews might draw attention away from the actual work of teaching (Ball, 2017). In the presentations in TSG-32, different methods for studying knowledge in and for teaching mathematics — more or less close to the work of teaching — were explored. Based on this exploration, a suggestion for future research might be similar to what Hoover et al. (2016) conclude based on their review, namely to "use sites where professional deliberation about teaching are taking place as sites where we might productively research the work of teaching and its mathematical demands" (p. 23). A lot of work remains to develop methodologies for studying mathematical knowledge in/for teaching through the work of teaching mathematics. Future papers, posters and discussions in TWG-32 at future ICME conferences will be an important site for discussing and developing such methodologies.

References

D. L. Ball (2017). Uncovering the special mathematical work of teaching. In G. Kaiser (Ed.), *Proceedings of the 13th International Congress on Mathematical Education* (pp. 11–34). Springer.

M. Hoover, R. Mosvold, D. L. Ball and Y. Lai (2016). Making progress on mathematical knowledge for teaching. *The Mathematics Enthusiast, 13*(1–2), 3–34.

L. S. Shulman (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.